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10MAT31

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. For the function :

$$f(x) = \begin{cases} x & \text{in} \quad 0 < x < \pi \\ x - 2\pi & \text{in} \quad \pi < x < 2\pi \end{cases}$$

Find the Fourier series expansion and hence deduce the result $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \text{-------}$. **(07 Marks)**

b. Obtain the half range Fourier cosine series of the function $f(x) = x(\ell - x)$ in $0 \le x \le \ell$. **(06 Marks)**

c. Find the constant term and first harmonic term in the Fourier expansion of y from the following table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

2 a. Find the Fourier transform of the function :

$$f(x) = \begin{cases} 1 & \text{for} \quad |x| \le a \\ 0 & \text{for} \quad |x| > a \end{cases}$$ and hence evaluate : $\int_0^\infty \frac{\sin x}{x} dx$. **(07 Marks)**

b. Obtain the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx, m > 0$. **(06 Marks)**

c. Solve the integral equation : $\int_0^\infty f(x) \cos px dx = \begin{cases} 1-p, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases}$ and hence deduce the value of $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. **(07 Marks)**

3 a. Obtain the various possible solutions of the two dimensional Laplace's equation $u_{xx} + u_{yy} = 0$ by the method of separation of variables. **(07 Marks)**

b. A string is stretched and fastened to two points 'ℓ' apart. Motion is started by displacing the string in the form $y = a \sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at time t = 0. Show that the displacement of any point at a distance 'x' from one end at time 't' is given by $y(x, t) = a \sin\left(\frac{\pi x}{\ell}\right) \cos\left(\frac{\pi ct}{\ell}\right)$. **(06 Marks)**

c. Obtain the D' Alembert's solution of the wave equation $u_{tt} = c^2 u_{xx}$ subject to the conditions $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = a$. **(07 Marks)**

4 a. For the following data fit an exponential curve of the form $y = a e^{bx}$ by the method of least squares :

x	5	6	7	8	9	10
y	133	55	23	7	2	2

(07 Marks)

b. Solve the following LPP graphically :
Minimize $Z = 20x + 10y$
Subject to the constraints : $x + 2y \leq 40$
$3x + y \geq 30$
$4x + 3y \geq 60$
$x \geq 0$ and $y \geq 0$. **(06 Marks)**

c. Using Simplex method, solve the following LPP :
Maximize : $Z = 2x + 4y$
Subject to the constraints $3x + y \leq 22$
$2x + 3y \leq 24$
$x \geq 0$ and $y \geq 0$. **(07 Marks)**

PART – B

5 a. Using the Regula – Falsi method to find the fourth root of 12 correct to three decimal places. **(07 Marks)**

b. Apply Gauss – Seidal method, to solve the following of equations correct to three decimal places :
$6x + 15y + 2z = 72$
$x + y + 54z = 110$
$27x + 6y - z = 8.5$
(carry out 3 iterations). **(06 Marks)**

c. Using Rayleigh power method, determine the largest eigen value and the corresponding eigen vector, of the matrix A in six iterations. Choose $[1\ 1\ 1]^T$ as the initial eigen vector :

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$ **(07 Marks)**

6 a. Using suitable interpolation formulae, find y(38) and y(85) for the following data :

x	40	50	60	70	80	90
y	184	204	226	250	276	304

(07 Marks)

b. If $y(0) = -12$, $y(1) = 0$, $y(3) = 6$ and $y(4) = 12$, find the Lagrange's interpolation polynomial and estimate y at $x = 2$. **(06 Marks)**

c. By applying Weddle's rule, evaluate : $\int_0^1 \frac{x dx}{1+x^2}$ by considering seven ordinates. Hence find the value of $\log_e^2$. **(07 Marks)**

7 a. Using finite difference equation, solve $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to u(0, t) = u(4, t) = 0, $u_t(x, 0) = 0$ and u(x, 0) = x(4 – x) upto four time steps. Choose h = 1 and k = 0.5. **(07 Marks)**

b. Solve the equation $u_t = u_{xx}$ subject to the conditions u(0, t) = 0, u(1, t) = 0, u(x, 0) = sin (π x) for $0 \le t \le 0.1$ by taking h = 0.2. **(06 Marks)**

c. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown. Find the first iterative values of $u_i (i = 1 - 9)$ to the nearest integer. **(07 Marks)**

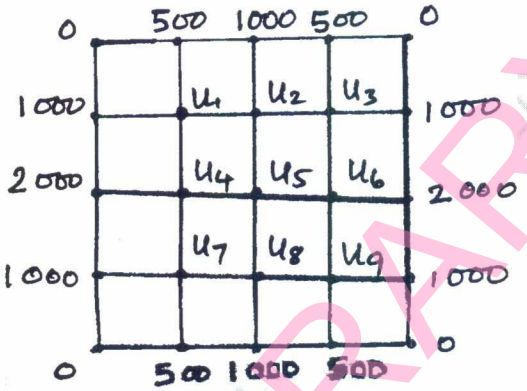

Fig.Q7(c)

8 a. Find the z – transform of 2n + sin (nπ/4) + 1. **(07 Marks)**

b. Obtain the inverse z – transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(06 Marks)**

c. Using z – transform, solve the following difference equation :
$u_{n+2} + 2u_{n+1} + u_n = n$ with $u_0 = u_1 = 0$. **(07 Marks)**

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10ME32B/AU32B

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016
Mechanical Measurements and Metrology

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART - A

1 a. What is Metrology? State any four objectives of metrology. **(06 Marks)**
b. Describe with a neat sketch, International Prototype meter. **(06 Marks)**
c. Discuss the important features of wavelength standard. **(04 Marks)**
d. Build 49.3115mm using M112 set of slip gauges. **(04 Marks)**

2 a. Define the following terms :
i) Limits ii) Fits iii) Tolerance iv) Deviation. **(04 Marks)**
b. Differentiate between interchangeability and selective assembly. **(06 Marks)**
c. Explain with a sketch, Taylor's principle for design of limit gauges. **(06 Marks)**
d. What is Wear allowance? How it is applied in design of gauges? **(04 Marks)**

3 a. What are the required characteristics of comparators? **(04 Marks)**
b. With neat sketch, describe the construction and principle of working of sigma comparator. **(12 Marks)**
c. Give the combination of angle gauges to obtain 57^0 34' 9" angle. **(04 Marks)**

4 a. Illustrate the principle of Interferrometry, with sketches. **(06 Marks)**
b. What is best wire size? Derive an expression for the same. **(06 Marks)**
c. With a sketch, explain the construction of a tool maker's microscope. What are its applications? **(08 Marks)**

PART - B

5 a. What is the significance of measurement system? **(04 Marks)**
b. Define the following terms used with reference to measurement :
i) Accuracy ii) Precision iii) Calibration iv) Threshold v) Sensitivity
vi) Hysteresis. **(06 Marks)**
c. Mention any four mechanical and four electrical transducers. **(04 Marks)**
d. Define an error. How errors are classified? Give reasons for each type of error during measurement. **(06 Marks)**

6 a. Describe in detail a ballast circuit. **(08 Marks)**
b. With a sketch, explain the construction and important parts of a cathode ray oscilloscope. **(08 Marks)**
c. What are the primary functions of a terminating device? **(04 Marks)**

7 a. Explain with a sketch, working of proving ring. **(06 Marks)**
b. Explain the working principle of hydraulic dynamometer used for torque measurement. **(08 Marks)**
c. Describe with a neat sketch, McLeod Vaccum gage. **(06 Marks)**

8 a. What is a Thermocouple? State the laws of thermocouple. **(08 Marks)**
b. Describe the construction and working of optical pyrometer. **(08 Marks)**
c. Write short note on gauge factor. **(04 Marks)**

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10ME/AU33

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016
Basic Thermodynamics

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. ***Use of thermodynamic data hand book and steam tables is permitted.***

PART – A

1 a. Differentiate between: (i) Control mass and control volume (ii) Intensive and extensive properties, and classify the following into intensive and extensive properties. (i) Enthalpy (ii) Quality of steam (iii) Entropy and (iv) Density. **(06 Marks)**

b. Define the zeroth law of thermodynamics and explain how this law forms the basis for temperature measurement. **(06 Marks)**

c. A constant volume gas thermometer containing helium gas gives readings of gas pressure 1000 mm and 1366 mms of mercury at the ice point and steam point respectively.
i) Express the gas thermometer Celsius temperature interms of gas pressure.
ii) The thermometer when left standing in the atmosphere, registers 1075 mm of mercury. Determine the atmospheric temperature. **(08 Marks)**

2 a. Write the thermodynamic definition of work. With suitable example explain how it is more general than the definition of work in mechanics. **(04 Marks)**

b. A gas expands quasi-statically in a piston cylinder arrangement against the atmosphere and a spring. Initial pressure and volume are 400 KPa and 0.2 m^3 respectively. The final volume of gas is 0.6 m^3. Determine the total work done by the gas if the spring force is proportional to volume of the gas. Assume the atmospheric pressure as 101.325 KPa. **(08 Marks)**

c. A spherical balloon of 1 m diameter contains a gas at 200 KPa. The gas inside the balloon is heated until the pressure reaches 500 KPa. During the process of heating, the pressure of the gas inside the balloon is proportional to the diameter of the balloon. Calculate the work done by the gas inside the balloon. **(08 Marks)**

3 a. Write the first law equation for a closed system undergoing a non cyclic process and show that internal energy is a property of a system. **(08 Marks)**

b. Modify the general steady flow energy equation (SFEE) for the following cases:
(i) Steam turbine with negligible potential energy change if the process is adiabatic.
(ii) Horizontal steam nozzle with negligible entrance velocity of steam, if the process is non-adiabatic.
(iii) Insulated horizontal throttle valve. **(06 Marks)**

c. In a centrifugal compressor, the suction and delivery pressures are 100 KPa and 550 KPa respectively. The compressor draws 15 m^3/min of air which has a specific volume of 0.77 m^3/kg. At the delivery the specific volume is 0.20 m^3/kg. The compressor is driven by a 40 KW motor, and heat lost to the surroundings during compression is 30 KJ/kg of air. Neglecting change in potential and kinetic energy, calculate increase in internal energy per kg of air. **(06 Marks)**

4 a. Write Kelvin-Planck and Clausius statements of second law of thermodynamics. Show that violation of Clausius statement leads to the possibility of a perpetual motion machine of second type. **(10 Marks)**

b. Mention the factors that make a process irreversible. **(04 Marks)**

c. Using a heat engine of thermal efficiency 30% to drive a refrigerator having a COP of 5, what is the heat received by the heat engine for each MJ of heat removed from the cold body of the refrigerator. **(06 Marks)**

PART – B

5 a. Derive Clausius unequality and prove that entropy is a property. **(10 Marks)**

b. Explain the principle of increase of entropy. **(04 Marks)**

c. Two copper blocks weighing 10 kg each are initially at temperatures of 227°C and 27°C respectively. What is the change in entropy when these two blocks are brought into contact with each other? Take specific heat of copper as 0.4 KJ/kgK. **(06 Marks)**

6 a. Define the following: (i) Pure substance (ii) Triple point (iii) Critical point. **(06 Marks)**

b. With neat sketch explain the measurement of dryness fraction of steam by using throttling calorimeter. **(08 Marks)**

c. Calculate the internal energy per kg of superheated steam at pressure of 10 bar and a temperature of 300°C. Also find the change in internal energy if this steam is expanded to 1.4 bar and dryness fraction 0.8. **(06 Marks)**

7 a. Starting from the relation $Tds = du + Pdv$, show that for an ideal gas undergoing a reversible adiabatic process, the law for the process is given by $TV^{\gamma-1}$ =Constant. **(06 Marks)**

b. Clearly distinguish between ideal and real gases. **(04 Marks)**

c. A quantity of air at a pressure of 100 KPa, 27°C occupying a volume of 0.5 m^3 is compressed to a pressure of 500 KPa and volume of 0.12 m^3 according to the law PV^n = constant. Find (i) The value of index 'n' (ii) The mass of air (iii) Work transfer (iv) Heat transfer during the process (v) Change in entropy. **(10 Marks)**

8 a. Write the Vander Waals equation of state. In what ways, it is an improvement over the ideal gas equation. **(06 Marks)**

b. Explain the following:
i) Compressibility factor ii) Reduced properties iii) Law of corresponding states iv) Generalised compressibility chart. **(08 Marks)**

c. Determine the specific volume of hydrogen gas when its pressure is 60 bar and temperature is 100 K by using, (i) Compressibility chart (ii) Vander Waal's equation
Take for H_2 $T_C = 239.76^0$C , P_C = 12.92 bar, $a = 0.25105 \times 10^5$ $Nm^2/kgmol^4$, $b = 0.0262$ m^3/kgmole. **(06 Marks)**

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10ME/AU34

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016
Mechanics of Materials

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART - A

1 a. Define i) Proof stress ii) Proportionality limit iii) Principle of superposition iv) Hooke's law. **(04 Marks)**

b. Derive an expression for the total elongation of the tapered bar varying diameter from d_1 to d_2, when subjected to axial load P. **(08 Marks)**

c. A brass bar having of uniform cross – sectional area of 300mm^2 is subjected to a load as shown below. Find the total elongation of bar and the magnitude of load 'P' if Young's modulus is 84 GPa. **(08 Marks)**

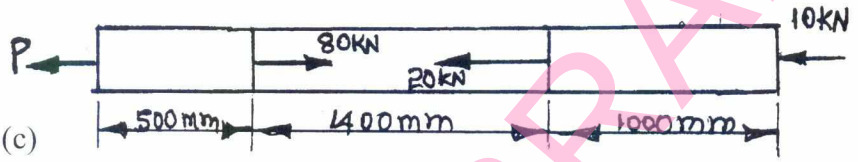

Fig.Q1(c)

2 a. Define : i) Poisson's ratio ii) Modular ratio. **(02 Marks)**

b. Establish relationship between Young's modulus and rigidity modulus. **(08 Marks)**

c. For the stepped bar shown in fig.Q2(c), what is the maximum temperature rise which will not produce stress in the bar. Also find the stress induced when the temperature rise is 40^0C. Take E_S = 200GPa ; E_A = 100GPa ; $\alpha_S = 12 \times 10^{-6}/^0$C ; $\alpha_A = 18 \times 10^{-6}/^0$C. **(10 Marks)**

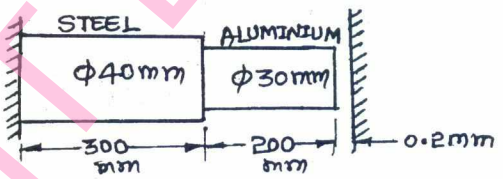

Fig.Q2(c)

3 a. Define : i) Plane stress ii) Principal strain. **(02 Marks)**

b. Derive the expressions for normal stress, shear stress and resultant stress on a oblique plane inclined at an angle 'θ' with vertical axis in a biaxial direct stress system. **(08 Marks)**

c. At a certain point in a strained material the values of normal stresses across two planes at right angles to each other are 80MPa and 32 MPa, both are tensile and there is a shear stress of 32MPa clockwise on the plane carrying 80MPa stress. Determine principal stresses, maximum shear stress and their planes. **(10 Marks)**

4 a. Determine the strain energy in a cantilever beam of uniform cross – section and length 'L' subjected to a uniformly distributed load of 'W' kN/m over the entire span. **(04 Marks)**

b. For a thin cylindrical shell, the L/d ratio is 3 and its initial volume is 20m^3. The ultimate stress for the cylinder material is 200MPa. Determine the wall thickness, if it has to convey water under a head of 200m. Take F.O.S as 2. **(08 Marks)**

c. Calculate the maximum external to internal radius ratio for a thick cylinder with internal fluid pressure of 15MPa and maximum hoop stress is 60MPa. **(08 Marks)**

PART - B

5 a. Deduce the relationship between relating load (W), shear force (F) and bending moment (M). **(06 Marks)**

b. A beam ABCD is simply supported at B and C, 4.5m apart and overhanging parts AB and CD are 1.5m and 2m long respectively. The beam carries a uniformly distributed load of 10kN/m between A & C. There is a clock wise couple of 60kN-m at D. Then draw S.F and B.M diagrams and mark salient points. **(14 Marks)**

6 a. Enumerate the assumptions made in theory of pure bending. Write the bending equation with usual notations. **(06 Marks)**

b. A beam of an I – section consists of 180mm × 15mm flanges and a web of 280mm × 15mm thickness. It is subjected to a bending moment of 120kN – m. Sketch the bending stress distribution along the depth of the section. **(06 Marks)**

c. Prove that in case of a rectangular section of a beam the maximum shear stress is 1.5 times average shear stress. **(08 Marks)**

7 a. Derive an expression $EI\frac{d^2y}{dx^2} = M$, with usual notations. **(10 Marks)**

b. Determine the deflection under the loads in the beam shown in fig.Q7(b). Take Flexural rigidity as EI, throughout. **(10 Marks)**

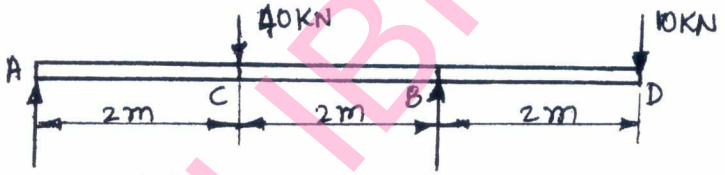

Fig.Q7(b)

8 a. A hollow circular steel shaft has to transmit 60KW at 210 rpm such that the maximum shear stress does not exceed 60MN/m^2. If the ratio of internal to external diameter is equal to ¾ and the value of rigidity modulus is 84GPa, find the dimensions of the shaft and angle of twist in a length of 3m. **(10 Marks)**

b. Derive an expression for the critical load in a column subjected to compressive load, when both ends are fixed. **(10 Marks)**

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10ME/AU35

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016

Manufacturing Process - I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Briefly Discuss the steps involved in making a sand casting. **(08 Marks)**

b. What is pattern? List different types of pattern. Explain match plate pattern with a neat sketch. **(09 Marks)**

c. Write explanatory note on No – Bake sands. **(03 Marks)**

2 a. Explain briefly the desirable properties of moulding sand. **(08 Marks)**

b. Draw a neat sketch of a gating system showing all the elements. **(04 Marks)**

c. With a neat sketch explain sand slinger. **(08 Marks)**

3 a. Explain with a neat sketch, the shell moulding process. **(10 Marks)**

b. With a neat sketch, explain continuous casting process. **(10 Marks)**

4 a. With a neat sketch, explain the different zones present in cupola furnace. **(10 Marks)**

b. With a neat sketch explain the constructional features and working of electrical resistance Furnace. List its advantages. **(10 Marks)**

PART – B

5 a. Sketch and explain TIG welding process. Mention its limitations. **(10 Marks)**

b. What are the different types of flames produced is oxy – acetylene welding and explain in brief with neat sketches. **(10 Marks)**

6 a. Explain the principle of Resistance welding. Also list its major applications. **(06 Marks)**

b. With a neat sketch explain laser beam welding. **(10 Marks)**

c. Enumerate the advantages of Electron beam welding. **(04 Marks)**

7 a. What is Heat Affected Zone (HAZ)? Explain the parameter affecting HAZ. **(10 Marks)**

b. List the functions of Electrode coatings. **(04 Marks)**

c. Write a short note on residual stresses in welding. **(06 Marks)**

8 a. Compare soldering and Brazing process. Mention their advantages, limitations and applications. **(12 Marks)**

b. Explain Radiography inspection with a neat sketch. **(08 Marks)**

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10ME/AU36B

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016
Fluid Mechanics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. $10m^3$ of mercury weighs 136×10^4N. Calculate its specific weight, mass density, specific volume and specific gravity. **(05 Marks)**

b. State Newton Law of viscosity, Classify different types of fluids. **(05 Marks)**

c. Derive an expression for a surface tension on liquid droplet. **(05 Marks)**

d. Derive on expression for a capillary fall, when the glass tube is dipped in mercury. **(05 Marks)**

2 a. State and explain Pascal and hydrodynamic laws. List the application of these laws. **(05 Marks)**

b. The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp gravity 0.9 is flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limb is 20cm. **(05 Marks)**

c. A Tank contains water upto a height of 0.5m above the base. An immiscible liquid of Sp. gravity 0.8 is filled on the top of water upto 1m height.
Calculate
i) Total pressure on one side of the tank ii) The position of centre of pressure for one side of the tank which is 2m wide. **(05 Marks)**

d. A rectangular plane surface 3m wide and 4m deep lies in water in such a way that its plane makes on angle of 30° with the free surface of water. Determine the total pressure force and position of centre of pressure, when the edge is 2m below the free surface. **(05 Marks)**

3 a. Derive an expression for the meta-centric height of a floating body. **(10 Marks)**

b. Classify the different types of Fluid Flow. **(05 Marks)**

c. The stream function for a two dimensional flow is given by $\psi = 2xy$. Calculate velocity at the point P(2, 3), find the velocity potential function ϕ. **(05 Marks)**

4 a. Derive Bernoulli's equation for the flow of an incompressible frictionless fluid from consideration of momentum. **(10 Marks)**

b. The water is flowing through a pipe having diameter 20cm and 10cm at section 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6m above datum and section 2 is 4m above datum. **(10 Marks)**

PART – B

5 a. A venturimeter is used for measurement of discharge of water in a horizontal pipe line. If the ratio of upstream pipe diameter to that of throat is 2:1 up stream diameter is 300mm, the difference of pressure between the throat and upstream is equal to 3m head of water and loss of head through meter is one eighth of the throat velocity head, calculate discharge in the pipe. **(10 Marks)**

b. Explain different types of forces acting in moving fluid. **(05 Marks)**

c. Explain the methods of Dimensional Analysis. **(05 Marks)**

6 a. How to determine the loss of head due to friction in pipes by using i) Darcy Formula ii) Chezy formula **(05 Marks)**

b. Explain the terms i) Major energy loss ii) Minor loss iii) Hydraulic gradient line iv) Total energy line. **(10 Marks)**

c. Find the diameter of a pipe of length 2000m when the rate of flow of water through the pipe is 200 ℓ/s and the head lost due to friction is 4m take the value of e = 50 in Chezy's Formula. **(05 Marks)**

7 a. Derive on expression for laminar flow through circular pipe [Hagen Poiseuille equation]. **(10 Marks)**

b. Derive on expression for laminar flow between two parallel stationary plates. **(10 Marks)**

8 a. Derive on expression for drag and lift. **(10 Marks)**

b. State the Bernoulli's theorem for compressible flow. Derive an expression for Bernoulli's equation when the process is i) Isothermal ii) Adiabatic process. **(10 Marks)**

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MATDIP301

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Express the following in the form a + ib, $\frac{3}{1+i}-\frac{1}{2-i}+\frac{1}{1-i}$ and also find the conjugate. **(06 Marks)**

b. Show that $(a+ib)^n+(a-ib)^n=2(a^2+b^2)^{n/2}\cos(n\tan^{-1}(b/a))$. **(07 Marks)**

c. Find the fourth roots of $1-i\sqrt{3}$ and represent them on an argand plane. **(07 Marks)**

2 a. Find the n^{th} derivative of cos 2x cos 3x. **(06 Marks)**

b. If $y=e^{a\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$. **(07 Marks)**

c. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. **(07 Marks)**

3 a. Find the angle between the radius vector and the tangent to the curve $r=a(1-\cos\theta)$ at the point $\theta=\frac{\pi}{3}$. **(06 Marks)**

b. Find the pedal equation to the curve $r=a(1+\cos\theta)$. **(07 Marks)**

c. Obtain the Maclaurin's series expansion of the function $e^x \sin x$. **(07 Marks)**

4 a. If $u=e^{x^3+y^3}$, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3u\log u$. **(06 Marks)**

b. If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$. **(07 Marks)**

c. If $u=x^2+y^2+z^2$, $v=xy+yz+zx$, $w=x+y+z$, find $J\left(\frac{u,v,w}{x,y,z}\right)$. **(07 Marks)**

5 a. Obtain the reduction formula for $I_n=\int_0^{\pi/2}\cos^n x\,dx$ where n is a positive integer. **(06 Marks)**

b. Evaluate : $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} xy\,dy\,dx$. **(07 Marks)**

c. Evaluate : $\int_0^1\int_0^1\int_0^1 (x+y+z)\,dx\,dy\,dz$. **(07 Marks)**

6 a. Prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

b. Evaluate: $\int_0^4 x^{3/2}(4-x)^{5/2}\,dx$. **(07 Marks)**

c. Evaluate: $\int_0^\infty x^6 e^{-3x}\,dx$. **(07 Marks)**

7 a. Solve: $\dfrac{dy}{dx}+x\sin 2y=x^3\cos^2 y$. **(06 Marks)**

b. Solve: $(e^y+y\cos xy)dx+(xe^y+x\cos xy)dy=0$. **(07 Marks)**

c. Solve: $x^2ydx-(x^3+y^3)dy=0$. **(07 Marks)**

8 a. Solve: $\dfrac{d^3y}{dx^3}-6\dfrac{d^2y}{dx^2}+11\dfrac{dy}{dx}-6y=0$. **(06 Marks)**

b. Solve : $(D^2-4)y=e^x+\sin 2x$. **(07 Marks)**

c. Solve : $(D^2+D+1)y=1+x+x^2$. **(07 Marks)**

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